Smoothed Particle Hydrodynamics

Techniques for the Physics Based Simulation of Fluids and Solids

Rigid Bodies

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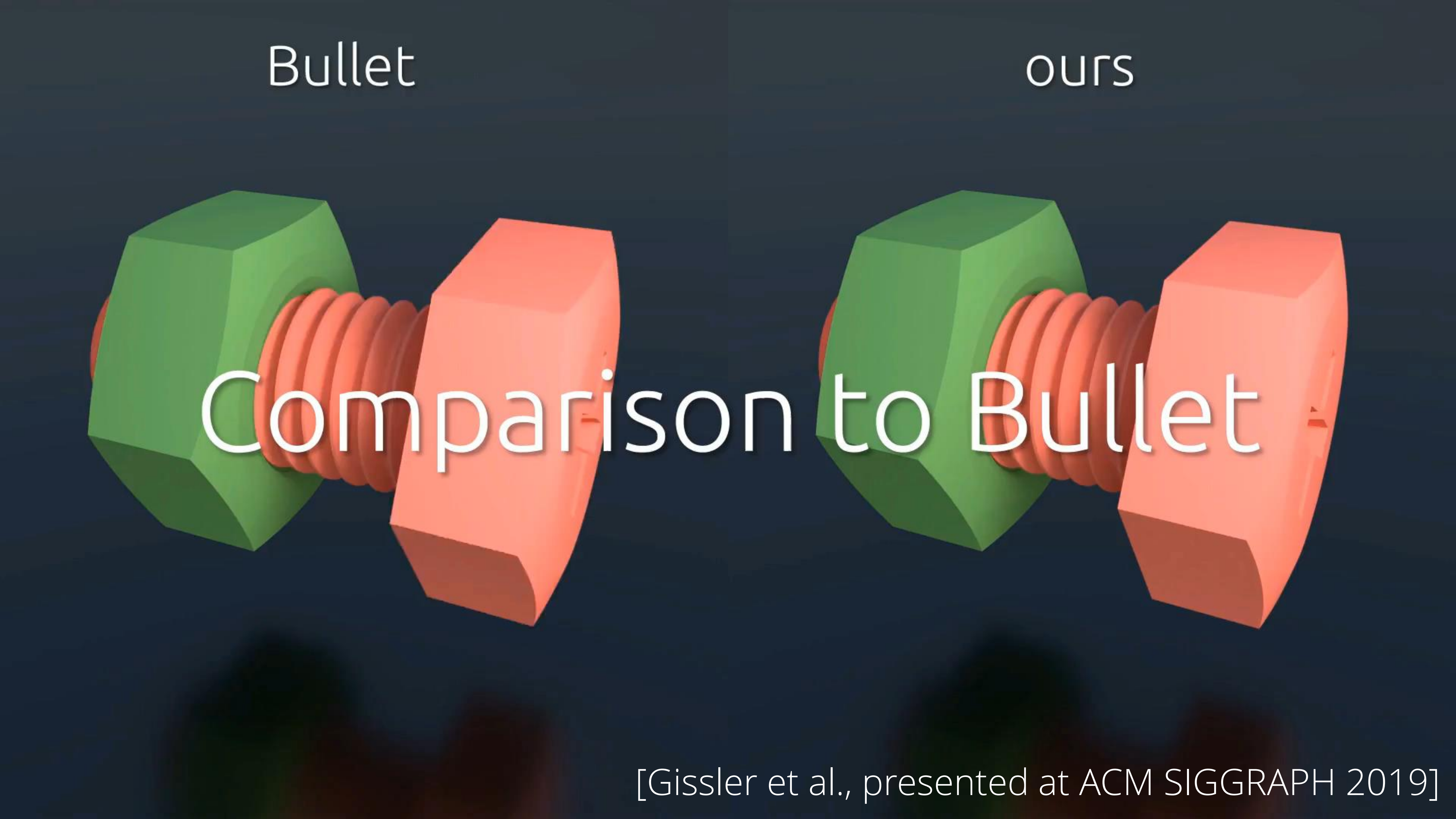
Graphics Research - SPH Solver

- Fluids
  - Low viscosity [Mueller 2003, Bender 2017]
  - High viscosity [Debrun 1996, Peer 15, Takahashi 15, Weiler 18]
  - Ferrofluids [Huang 2019]
- Granular materials
- Elastic solids [Solenthaler 2007, Peer 2018]
- Plastic solids [Gerszewski 2009]
- Rigid bodies [Gissler 2019]

SPH Rigid-Body Solver

- Sample surfaces with particles
- Map contact to artificial density deviation
- Compute pressure
- Pressure accelerations resolve contacts
- Pressure system differs from fluids as accelerations are applied to entire rigid bodies (sets of particles) instead of single particles


Bullet
ours
Comparison to Bullet
[Gissler et al., presented at ACM SIGGRAPH 2019]

Formulation

- Rigid-body surface particles with artificial rest density, e.g. $\rho_r^0 = 1$
- Contact: $\rho_r - \rho_r^0 > 0$
- Continuity equation: $\frac{\mathrm{D}\rho_r}{\mathrm{D}t} = -\rho_r \nabla \cdot \boldsymbol{v}_r$
- Time discretization at $t + \Delta t$:

$$\frac{\rho_r^{t+\Delta t} - \rho_r}{\Delta t} = -\rho_r \nabla \cdot \boldsymbol{v}_r^{t+\Delta t}$$

- Constraint $\rho_r^{t+\Delta t} = \rho_r^0$:

$$\frac{\rho_r^0 - \rho_r}{\Delta t} = -\rho_r \nabla \cdot \boldsymbol{v}_r^{t+\Delta t}$$

Concept

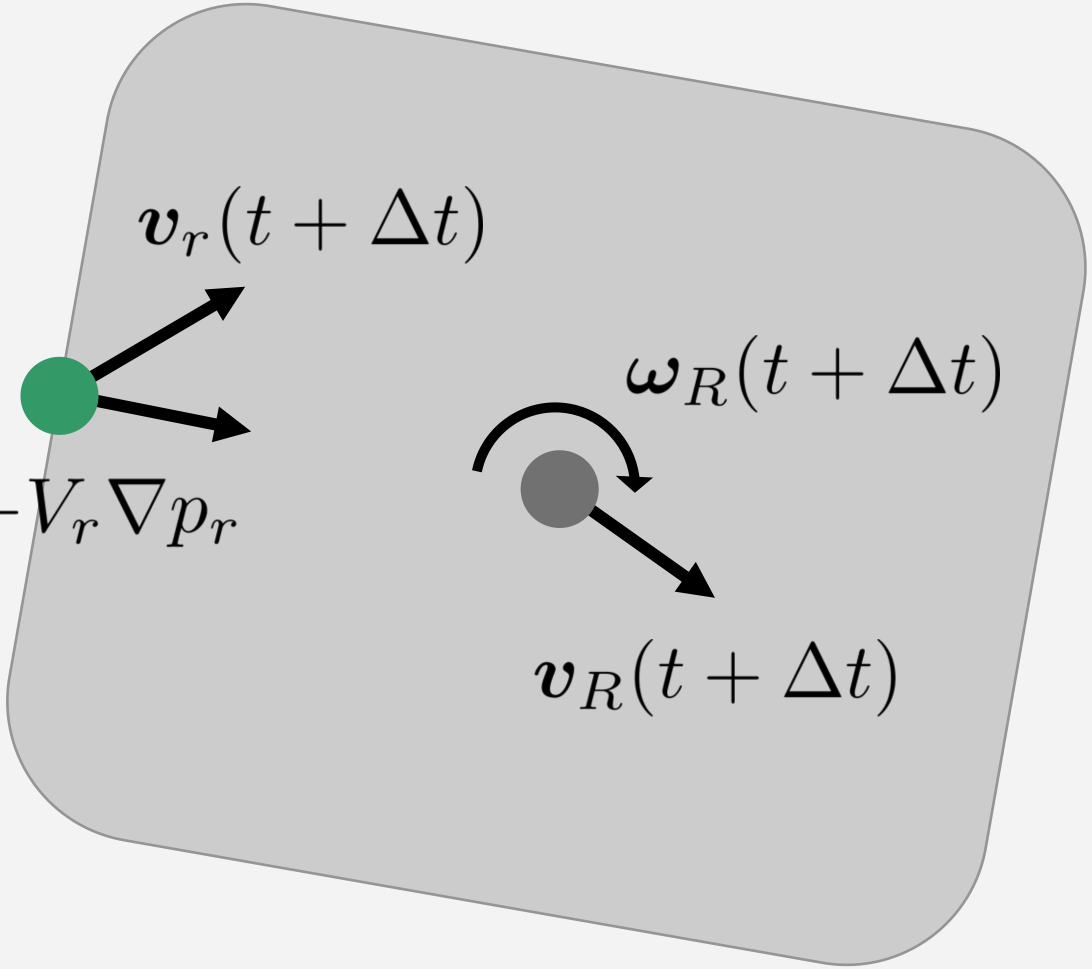
- Find velocities for rigid-body particles such that $\rho_r^0 = \rho_r - \Delta t \rho_r \nabla \cdot \boldsymbol{v}_r^{t+\Delta t}$
- Map unknown velocities $\boldsymbol{v}_r^{t+\Delta t}$ to unknown artificial pressure p_r per particle
- Pressure p_r corresponds to a pressure / contact force $\boldsymbol{F}_r^{rr} = -V_r \nabla p_r$
- Forces $\boldsymbol{F}_r^{rr}$ change linear / angular momentum of the entire rigid body such that all particles have the desired velocities $\boldsymbol{v}_r^{t+\Delta t}$

Illustration

$$\rho_r - \rho_r^0 > 0$$

$$\rho_r^0 = \rho_r - \Delta t \rho_r \nabla \cdot \boldsymbol{v}_r^{t+\Delta t}$$

$$\boldsymbol{F}_r^{rr} = -V_r \nabla p_r$$



Reformulating the System

- One equation per particle
  with unknown particle velocity

$$-\rho_r \nabla \cdot \boldsymbol{v}_r^{t+\Delta t} = \frac{\rho_r^0 - \rho_r}{\Delta t}$$

- One equation per particle
  with unknown body velocities

$$-\rho_r \nabla \cdot \left(\boldsymbol{v}_R^{t+\Delta t} + \boldsymbol{\omega}_R^{t+\Delta t} \times \boldsymbol{r}_r^t\right) = \frac{\rho_r^0 - \rho_r}{\Delta t}$$

$$\boldsymbol{v}_R^{t+\Delta t} = \boldsymbol{v}_R + \Delta t \frac{1}{M_R} \left(\boldsymbol{F}_R + \sum_k \boldsymbol{F}_k^{rr}\right)$$

$$\boldsymbol{\omega}_R^{t+\Delta t} = \boldsymbol{\omega}_R + \Delta t \boldsymbol{I}_R^{-1} \left(\tau_R + (\boldsymbol{I}_R \boldsymbol{\omega}_R) \times \boldsymbol{\omega}_R + \sum_k \boldsymbol{r}_k \times \boldsymbol{F}_k^{rr}\right)$$

Reformulating the System

- One equation per particle with unknown contact forces

$$-\rho_r \nabla \cdot \left(\Delta t \sum_k K_{rk} \boldsymbol{F}_k^{rr}\right) = \frac{\rho_r^0 - \rho_r}{\Delta t} \qquad K_{rk} = \frac{1}{M_R}\mathbb{1} - \tilde{\boldsymbol{r}}_r \boldsymbol{I}_R^{-1} \tilde{\boldsymbol{r}}_k$$

$$\boldsymbol{F}_r^{rr} = -V_r \nabla p_r$$

- One equation per particle with unknown pressures

$$\rho_r \nabla \cdot \left(\Delta t \sum_k V_k K_{rk} \nabla p_k\right) = \frac{\rho_r^0 - \rho_r}{\Delta t}$$

Solver

- SPH discretization of $\rho_r \nabla \cdot (\Delta t \sum_k V_k K_{rk} \nabla p_k)$
  - Two loops over all rigid particles
  - Standard SPH forms for $\nabla A_i, \nabla \cdot \boldsymbol{A}_i$
- Jacobi iterations

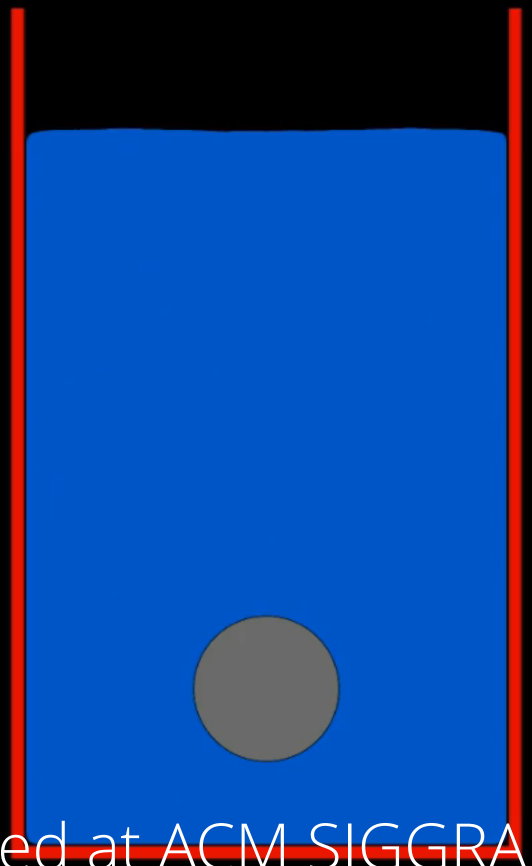
Strong Fluid-Rigid Coupling

- Iterative rigid-body solver
- Iterative fluid solver, e.g. PCISPH, IISPH, DFSPH
- Interleaved fluid-rigid velocity update
- Rigid solver iteration updates predicted velocities of rigid particles
- Fluid solver iteration updates predicted velocities of fluid particles

Rising sphere

Comparison to Akinci et al. 2012

our strong two-way coupling
allows to use a time step 100 times
larger compared to Akinci et al. 2012

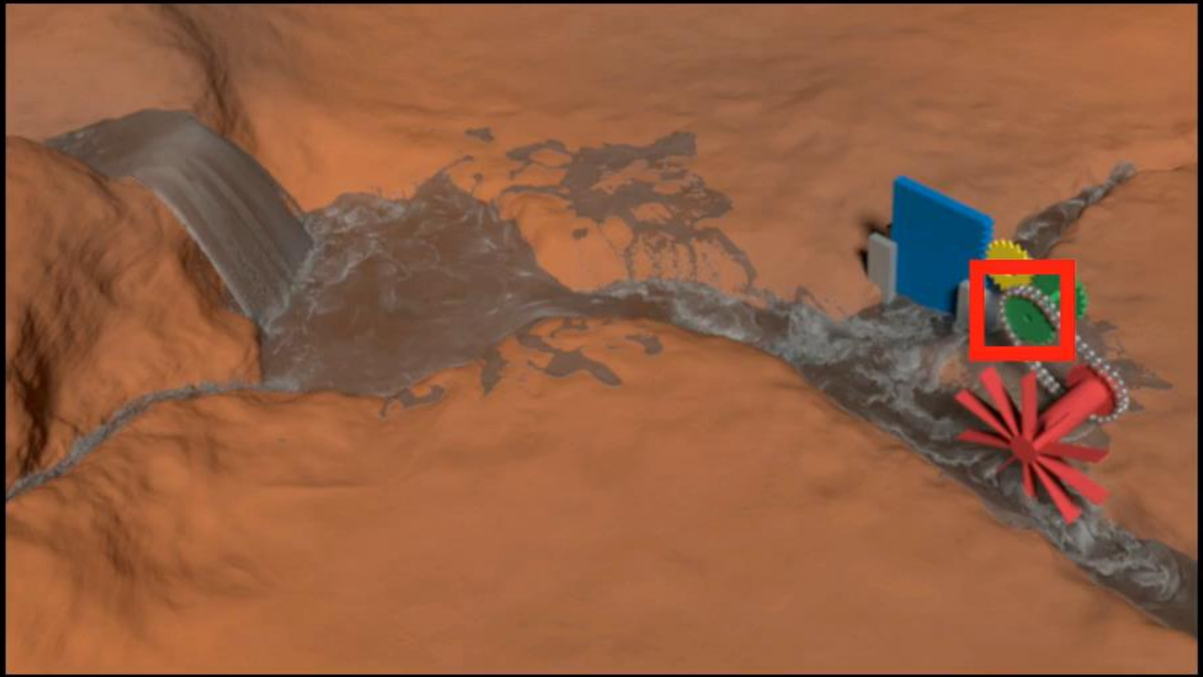


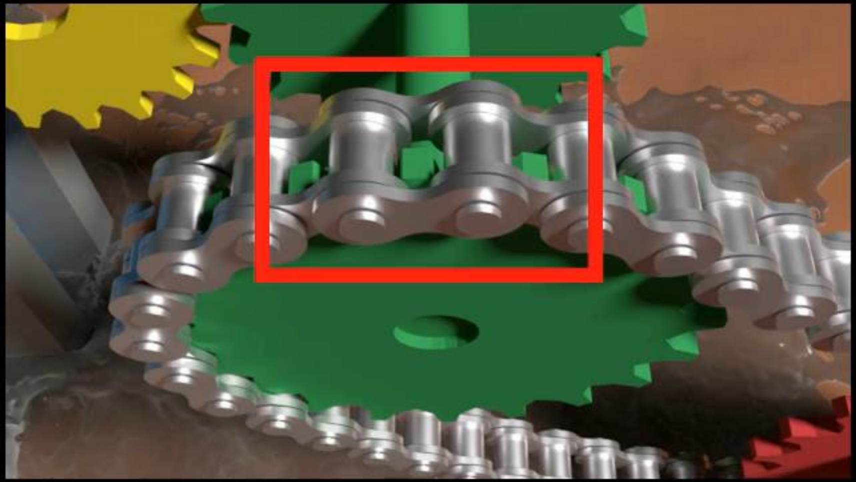
[Gissler et al., presented at ACM SIGGRAPH 2019]

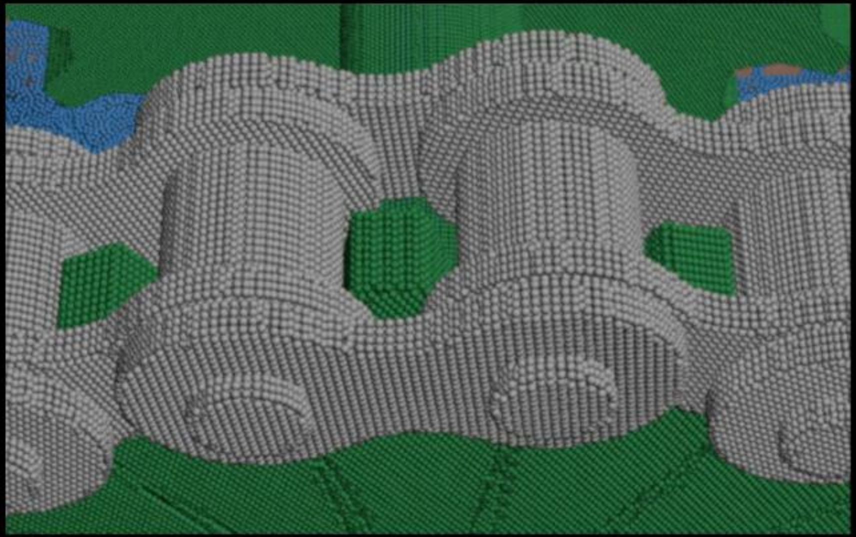

Propeller pump
propeller rotates with 160rpm
our approach stably simulates
fast objects with large time steps
[Gissler et al., presented at ACM SIGGRAPH 2019]

Water gate

chain elements are fully simulated using rigid-rigid contacts







up to 44M fluid particles
50M static rigid particles
2.3M dynamic rigid particles
up to 90k simultaneous rigid-rigid contacts

[Gissler et al., presented at ACM SIGGRAPH 2019]

PreonLab

v3.1

Showcase

Gearbox model kindly provided by Aniket Malbari [FIFTY2 Technology]