Smoothed Particle Hydrodynamics

Techniques for the Physics Based Simulation of Fluids and Solids

Boundary Handling

Dan Koschier

Jan Bender

Barbara Solenthaler

Matthias Teschner

UCL

RWTH

ETH zürich

UNI FREIBURG

SPH Fluid Solver

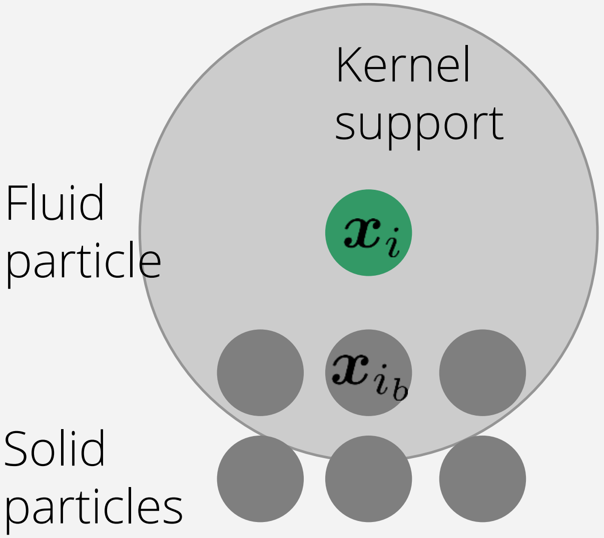
– Neighbor search
– Incompressibility
– Boundary handling

Outline

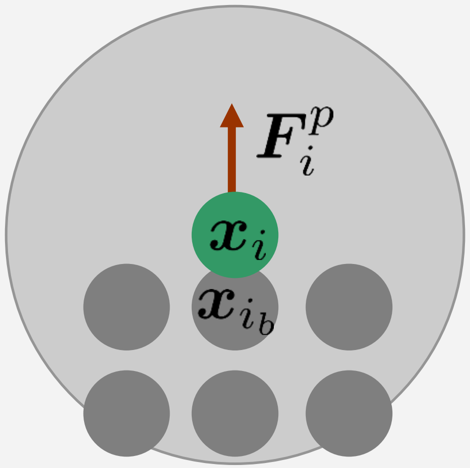
– Particle boundaries
– Current developments

Concept

– Boundaries are sampled with particles that contribute to density, pressure and pressure acceleration of the fluid



$$\rho_i < \rho_0$$
$$p_i = 0$$
$$p_{i_b} = 0$$
$$\boldsymbol{F}_i^{\mathrm{p}} = \boldsymbol{0}$$

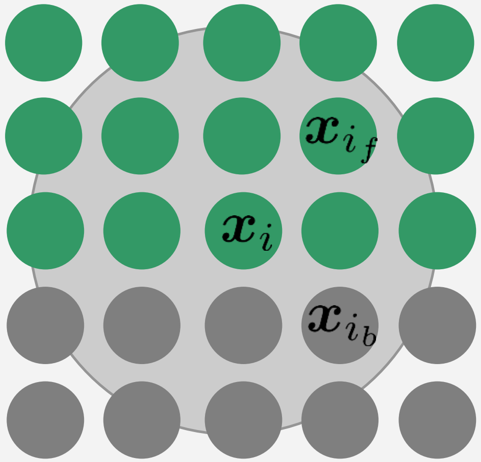


$$\rho_i > \rho_0$$
$$p_i > 0$$
$$p_{i_b} > 0$$
$$\boldsymbol{F}_i^{\mathrm{p}} \neq \boldsymbol{0}$$

– Boundary handling: How to compute $\rho_i, p_i, p_{i_b}, \boldsymbol{F}_i^{\mathrm{p}}$?

Several Layers with Uniform Boundary Samples

– Boundary particles are handled as static fluid samples



$$\rho_i = \sum_{i_f} m_{i_f} W_{ii_f} + \sum_{i_b} m_{i_b} W_{ii_b}$$

Boundary neighbors contribute to the density

$$m_i = m_{i_f} = m_{i_b}$$

All samples have the same size, i.e. same mass and rest density

$$\rho_i = m_i \sum_{i_f} W_{ii_f} + m_i \sum_{i_b} W_{ii_b}$$

$$p_i = k\left(\frac{\rho_i}{\rho_0} - 1\right)$$

– Pressure acceleration

$$\boldsymbol{a}_i^{\mathrm{p}} = -m_i \sum_{i_f} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_f}}{\rho_{i_f}^2} \right) \nabla W_{ii_f} - m_i \sum_{i_b} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_b}}{\rho_{i_b}^2} \right) \nabla W_{ii_b}$$

All samples have the same size, i.e. same mass and rest density

Contributions from fluid neighbors

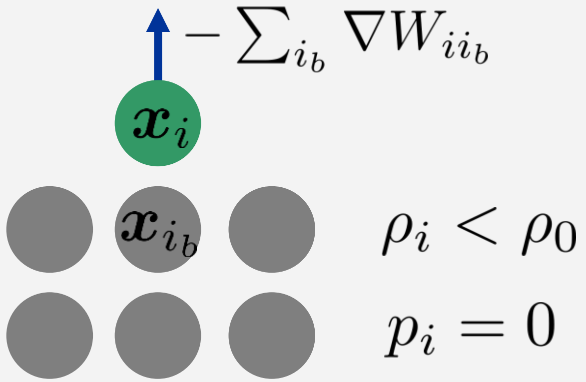
Contributions from boundary neighbors

Pressure at Boundary Samples

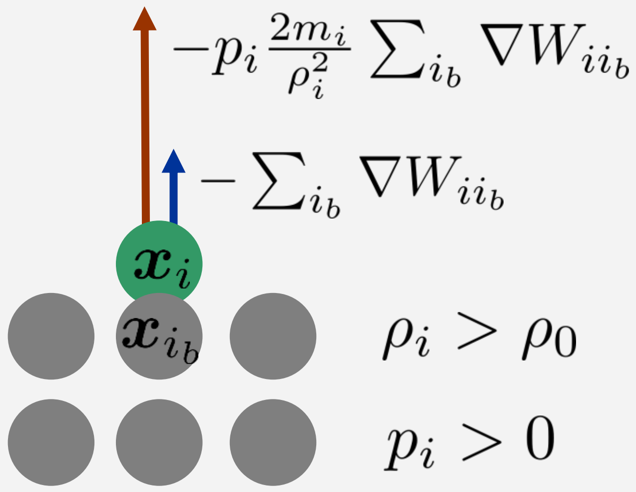
- Pressure acceleration at boundaries requires pressure at boundary samples
- Various solutions, e.g. mirroring, extrapolation, PPE
- Mirroring
  - Formulation with unknown boundary pressure p_{i_b}
  - $\boldsymbol{a}_i^{\mathrm{p}} = -m_i \sum_{i_f} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_f}}{\rho_{i_f}^2} \right) \nabla W_{ii_f} - m_i \sum_{i_b} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_b}}{\rho_{i_b}^2} \right) \nabla W_{ii_b}$
  - Mirroring of pressure and density from fluid to boundary $p_{i_b} = p_i$
  - $\boldsymbol{a}_i^{\mathrm{p}} = -m_i \sum_{i_f} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_f}}{\rho_{i_f}^2} \right) \nabla W_{ii_f} - m_i \sum_{i_b} \left(\frac{p_i}{\rho_i^2} + \frac{p_i}{\rho_i^2} \right) \nabla W_{ii_b}$

Boundary Contribution to Pressure Acceleration

$$\boldsymbol{a}_i^{\mathrm{p}} = - \ldots - m_i \sum_{i_b} \left(\frac{p_i}{\rho_i^2} + \frac{p_i}{\rho_i^2} \right) \nabla W_{ii_b} = - \ldots - p_i \frac{2m_i}{\rho_i^2} \sum_{i_b} \nabla W_{ii_b}$$



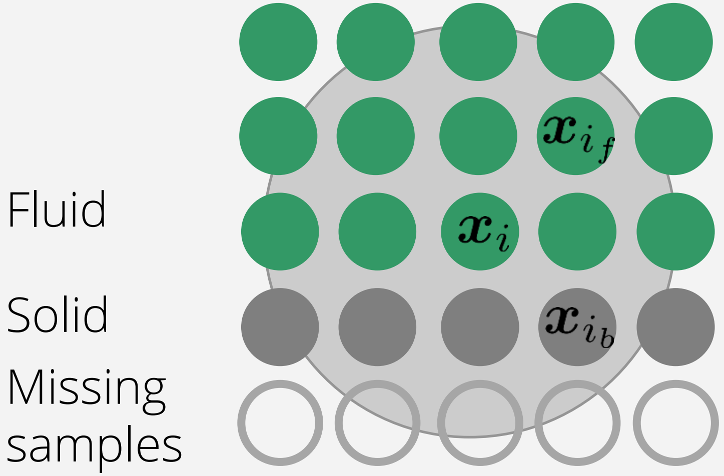
$$-p_i \frac{2m_i}{\rho_i^2} \sum_{i_b} \nabla W_{ii_b} = \mathbf{0}$$



$$-p_i \frac{2m_i}{\rho_i^2} \sum_{i_b} \nabla W_{ii_b} \neq \mathbf{0}$$

One Layer of Uniform Boundary Samples

– Contributions of missing samples have to be added



$$\rho_i = m_i \sum_{i_f} W_{ii_f} + m_i \sum_{i_b} W_{ii_b} + x$$

x is an approximation of the contribution from missing samples

$$\rho_i = m_i \sum_{i_f} W_{ii_f} + \gamma_1 m_i \sum_{i_b} W_{ii_b}$$

Offset typically implemen-ted as scaling coefficient

$$\sum_{i_f} W_{ii_f} + \gamma_1 \sum_{i_b} W_{ii_b} = \frac{1}{V_i} \;\Rightarrow\; \gamma_1 = \frac{\frac{1}{V_i} - \sum_{i_f} W_{ii_f}}{\sum_{i_b} W_{ii_b}}$$

Kernel property

– Pressure acceleration

$$\boldsymbol{a}_i^{\mathrm{p}} = -m_i \sum_{i_f} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_f}}{\rho_{i_f}^2} \right) \nabla W_{ii_f} - p_i \frac{2\gamma_2 m_i}{\rho_i^2} \sum_{i_b} \nabla W_{ii_b}$$

$$\sum_{i_f} \nabla W_{ii_f} + \gamma_2 \sum_{i_b} \nabla W_{ii_b} = \mathbf{0} \;\Rightarrow\; \gamma_2 = -\frac{\sum_{i_f} \nabla W_{ii_f} \cdot \sum_{i_b} \nabla W_{ii_b}}{\sum_{i_b} \nabla W_{ii_b} \cdot \sum_{i_b} \nabla W_{ii_b}}$$

Kernel gradient property
Pseudo inverse

Correction of Missing Contributions

$$\rho_i = m_0(W_{00} + W_{01} + W_{02})$$

$$\boldsymbol{a}_i^{\mathrm{p}} = -p_i \frac{2m_i}{\rho_i^2}(\nabla W_{01} + \nabla W_{02})$$

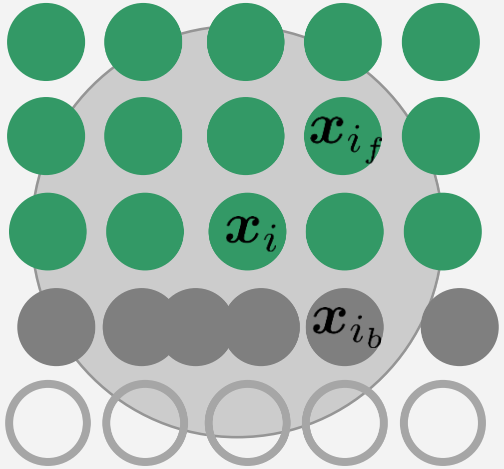
$$\rho_i = \gamma_1 m_0(W_{00} + W_{01})$$

$$\boldsymbol{a}_i^{\mathrm{p}} = -p_i \frac{2\gamma_2 m_i}{\rho_i^2}\nabla W_{01}$$

– The motivation of γ_1 and γ_2 is to compensate contributions of missing samples to $\rho, p, \boldsymbol{a}^{\mathrm{p}}$

One Layer of Non-Uniform Boundary Samples

– Non-uniform contributions from boundary samples



$$\rho_i = m_i \sum_{i_f} W_{ii_f} + \sum_{i_b} m_{i_b} W_{ii_b}$$

Non-uniform sizes, i.e. masses of boundary samples

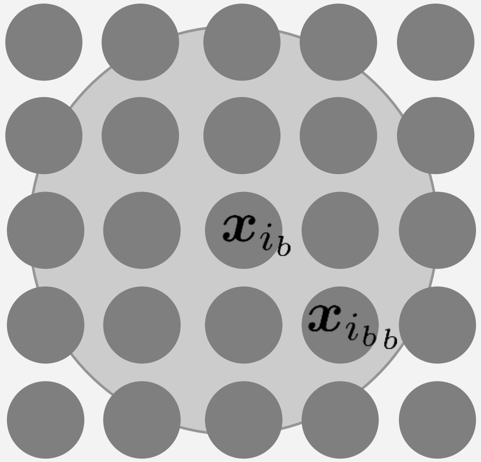
$$V_{i_b}^0 = \frac{m_{i_b}}{\rho_0} = \frac{\gamma_1}{\sum_{i_{b_b}} W_{i_b i_{b_b}}}$$

Contribution, i.e. mass of a boundary sample is approximated from its boundary neighbors

– Pressure acceleration

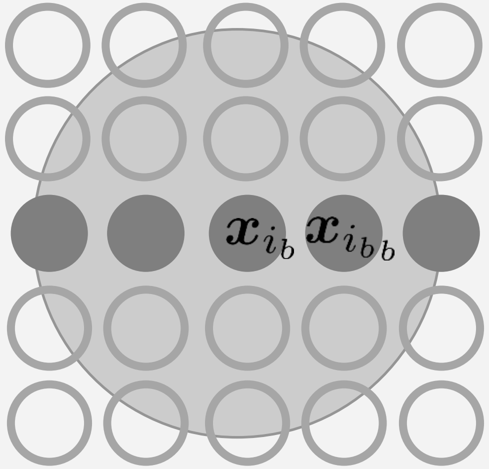
$$\boldsymbol{a}_i^{\mathrm{p}} = -m_i \sum_{i_f} \left(\frac{p_i}{\rho_i^2} + \frac{p_{i_f}}{\rho_{i_f}^2} \right) \nabla W_{ii_f} - p_i \frac{2\gamma_2}{\rho_i^2} \sum_{i_b} m_{i_b} \nabla W_{ii_b}$$

One Layer of Non-Uniform Boundary Samples



$$V_{i_b}^0 = h^3 = \frac{1}{\sum_{i_{bb}} W_{i_b i_{bb}}}$$

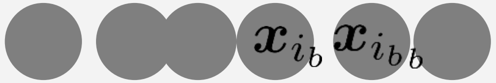
For perfect sampling



$$V_{i_b}^0 = h^3 = \frac{\gamma_1}{\sum_{i_{bb}} W_{i_b i_{bb}}}$$

$$\Rightarrow \gamma_1 = h^3 \sum_{i_{bb}} W_{i_b i_{bb}}$$

For perfect sampling

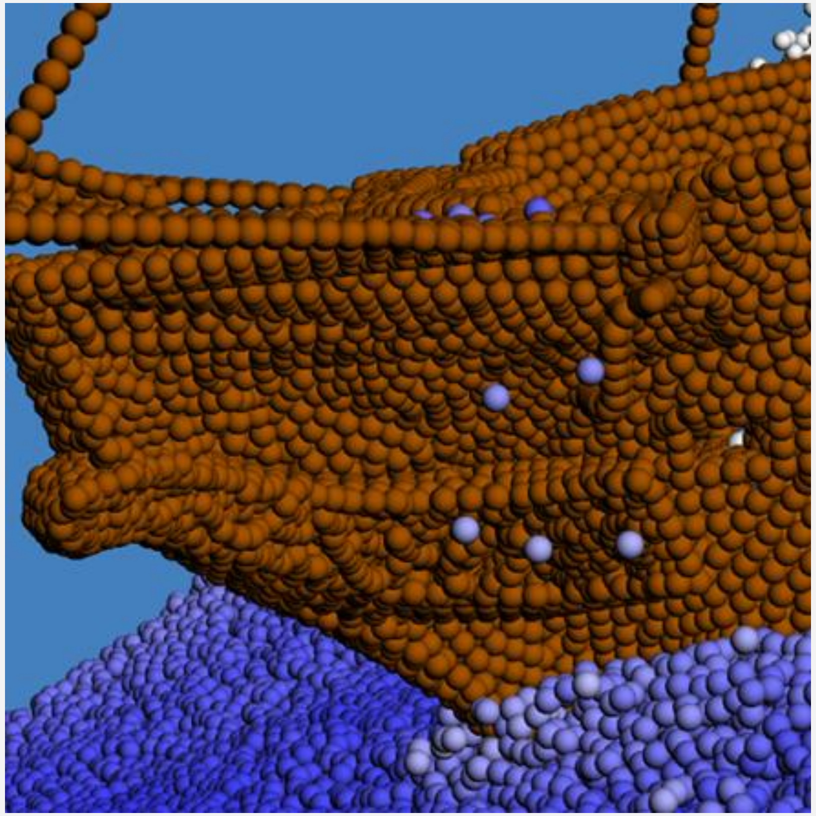


$$m_{i_b} = \rho_0 \frac{\gamma_1}{\sum_{i_{bb}} W_{i_b i_{bb}}}$$

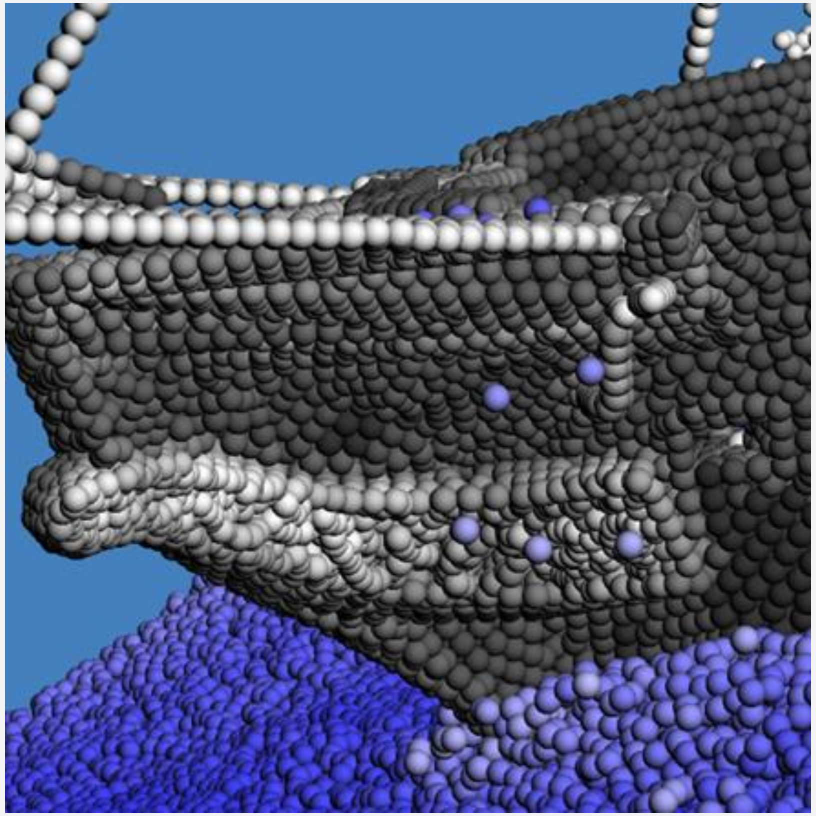
For arbitrary sampling

– In 3D, $\gamma_1 = 0.7$

Typical Boundary Representation

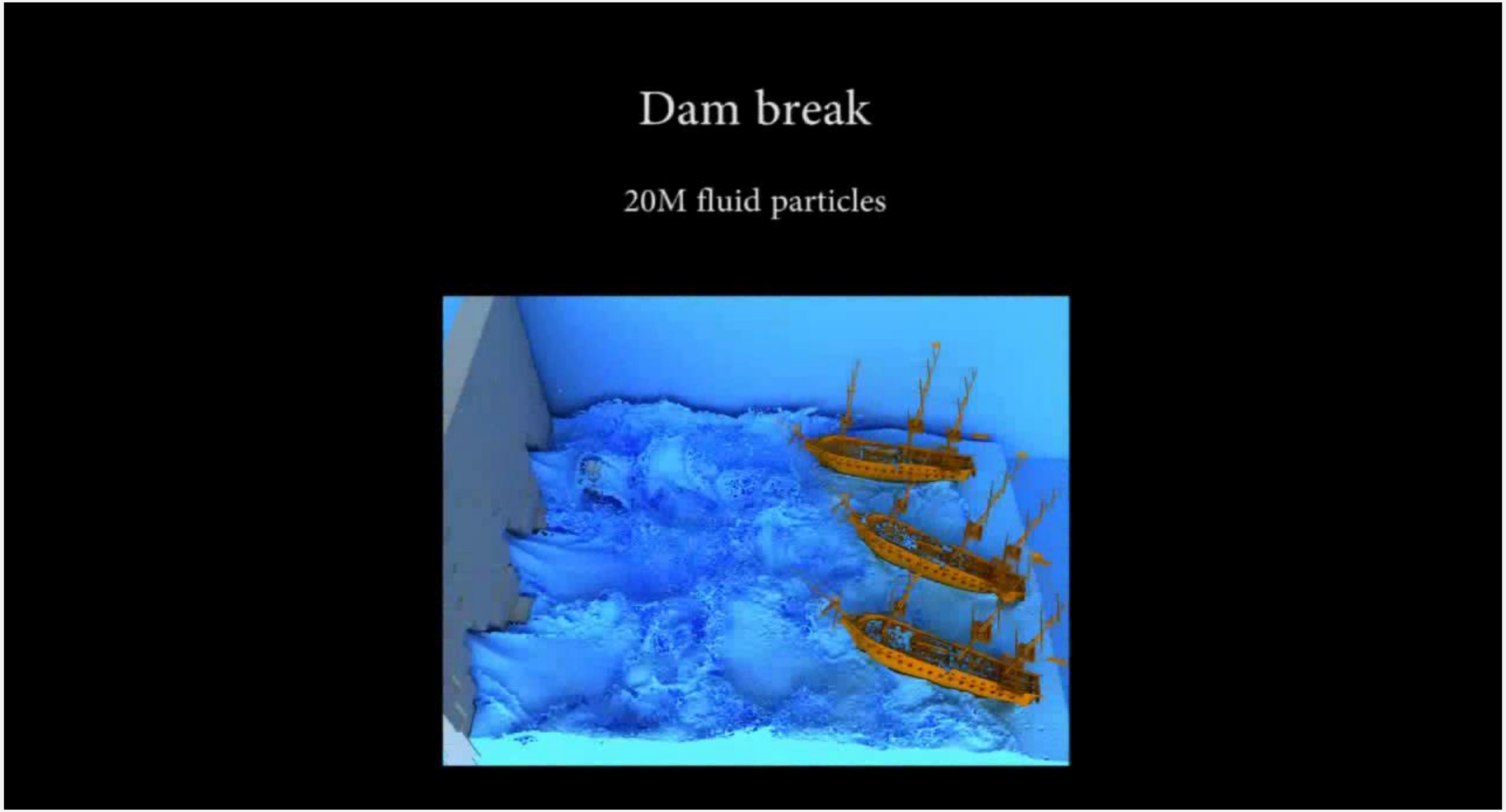


Boundary samples



Color-coded volume
of boundary samples

Rigid-Fluid Coupling



Rigid-Fluid Coupling

Summary

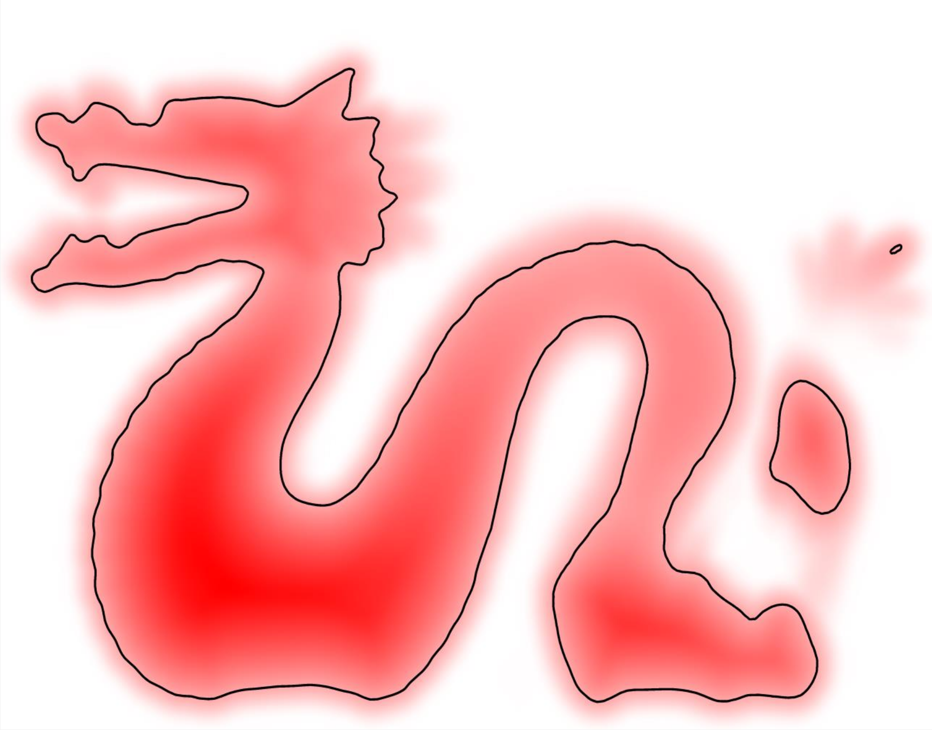
- Boundary is sampled with static fluid particles
- One layer of non-uniform samples
  - Arbitrary triangulated meshes can be used as boundary
  - Non-uniform boundary samples can be handled
  - Missing contributions to fluid density and pressure acceleration have to be corrected
  - Pressure is mirrored from fluid to boundary

Outline

– Particle boundaries
– Current developments

Current Developments

– Pressure extrapolation [Adami 2012, Band 2018]
– Solving boundary pressure with a PPE [Band 2018]
– Density maps [Koschier 2017]
    – Precomputing boundary contributions to the density computation of nearby fluid particles



SPH Fluid Solver

- Neighbor search
- Incompressibility
- Boundary handling