Smoothed Particle Hydrodynamics Techniques for the Physics Based Simulation of Fluids and Solids

VORTICITY

Dan Koschier Jan Bender Barbara Solenthaler Matthias Teschner

Outline

- Motivation
- Vorticity Confinement
- Micropolar Model
- Results

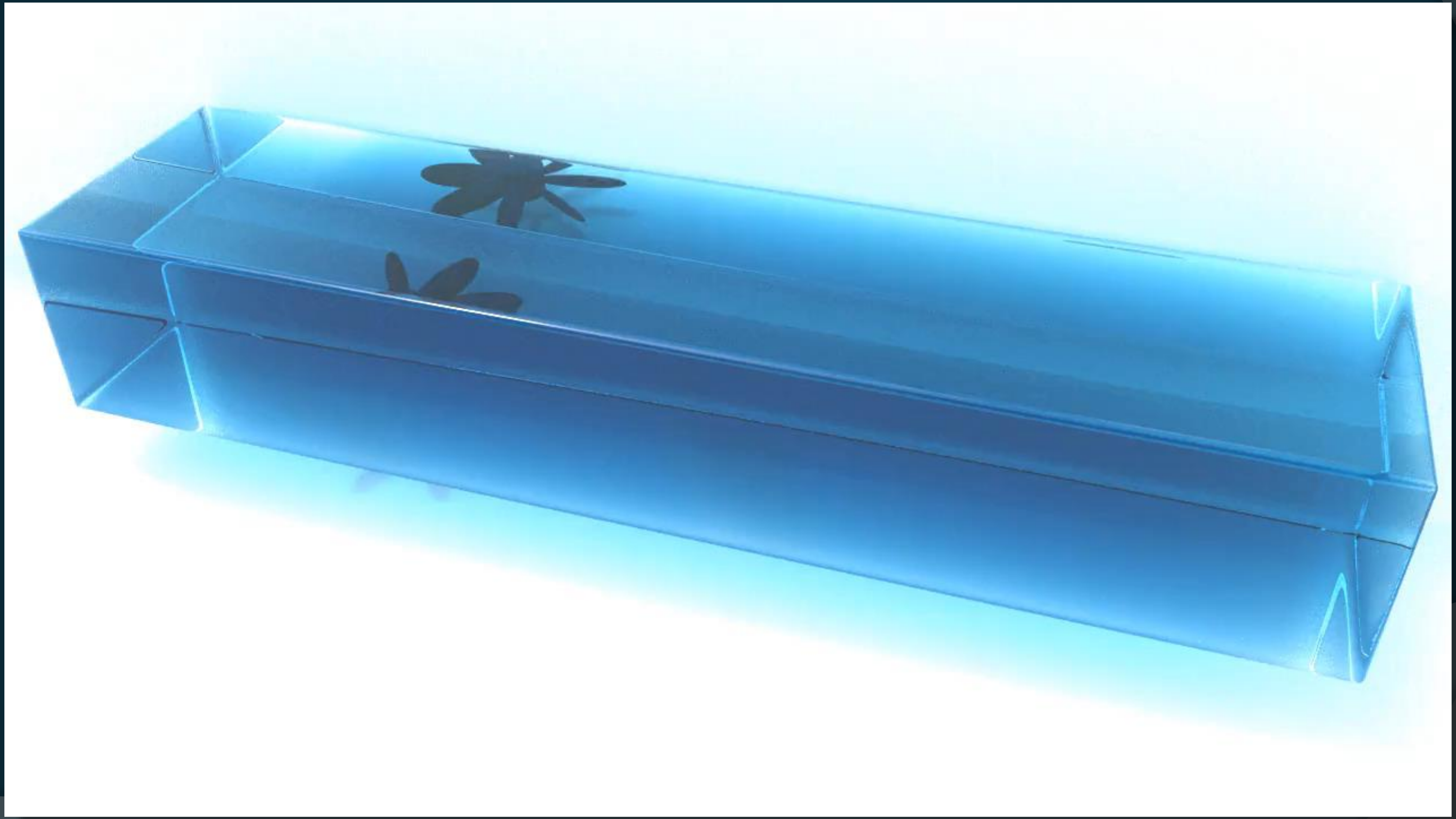
Motivation

- Turbulences in dynamic fluids is a visually important phenomenon.
- Turbulent motion is caused by the interaction of vortices.
- A vortex is a local spinning motion in the fluid which is defined as

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}$$

- Known SPH issue: turbulent details get lost due to numerical damping which negatively influences the visual liveliness.

Motivation



Outline

- Motivation
- Vorticity Confinement
- Micropolar Model
- Results

Vorticity Confinement

- Main idea: counteract the dissipation by amplifying existing vortices
- Vorticity confinement consists of three steps:
  1. Compute vorticity for each particle:

$$\boldsymbol{\omega}_i = \nabla \times \mathbf{v}_i = -\sum_j \frac{m_j}{\rho_j}(\mathbf{v}_i - \mathbf{v}_j) \times \nabla W_{ij}$$

Vorticity Confinement

2. Amplify existing vortices by a corrective force

$$\mathbf{F}_i^{\text{vorticity}} = \varepsilon^{\text{vorticity}} \left(\frac{\eta}{\|\eta\|} \times \boldsymbol{\omega}_i \right)$$

using the vorticity location vector

$$\eta = \sum_j \frac{m_j}{\rho_j} \|\boldsymbol{\omega}_j\| \nabla W_{ij}$$

3. Smooth velocity field using XSPH to get a coherent particle motion.

Outline

- Motivation
- Vorticity Confinement
- Micropolar Model
- Results

Micropolar Model

- Fluid particles with a microstructure
- Non-symmetric stress tensor
- Additional microrotation / angular velocity field
  - Vortices can exist independently of the linear velocity field
  - Angular velocity is less affected by numerical damping
  - Microrotation acts as additional source of vorticity
- Micropolar fluid model produces more realistic turbulences

Classical Newtonian Fluid Model

Linear momentum equation:

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \mathbf{T} + \mathbf{f}$$

- Density ρ
- Velocity $\mathbf{v}$
- Stress tensor $\mathbf{T}$
- Force density $\mathbf{f}$

Newtonian constitutive model:

$$\mathbf{T} = -p\mathbb{1} + \mu\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right)$$

- Pressure p
- Identity $\mathbb{1}$
- Dynamic viscosity μ

Navier-Stokes equations: $$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu\Delta\mathbf{v} + \mathbf{f}$$

Micropolar Model

Linear momentum equation:

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \mathbf{T} + \mathbf{f}$$

- Density ρ
- Velocity $\mathbf{v}$
- Stress tensor $\mathbf{T}$
- Force density $\mathbf{f}$

Angular momentum equation:

$$\rho\Theta \frac{D\boldsymbol{\omega}}{Dt} = \mathbf{T}_\times + \boldsymbol{\tau}$$

- Microinertia coeff. Θ
- Angular velocity $\boldsymbol{\omega}$
- $[\mathbf{T}_\times]_i = \sum_j \sum_k \epsilon_{ijk} T_{jk}$
- External torque density $\boldsymbol{\tau}$

Micropolar constitutive model for inviscid fluids:

$$\mathbf{T} = -p\mathbb{1} - \mu_t \nabla \mathbf{v} + \mu_t \boldsymbol{\omega}^\times$$

- Dynamic transfer param. μ_t
- Skew symmetric matrix $\boldsymbol{\omega}^\times$

Micropolar equations of motion:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu_t \nabla \times \boldsymbol{\omega} + \mathbf{f} \qquad \rho\Theta \frac{D\boldsymbol{\omega}}{Dt} = \mu_t(\nabla \times \mathbf{v} - 2\boldsymbol{\omega}) + \boldsymbol{\tau}$$

Micropolar Model

- Final equations of motion:

$$\rho \frac{D\mathbf{v}}{Dt} = \boxed{-\nabla p} + \boxed{\mu_t \nabla \times \boldsymbol{\omega}} + \boxed{\mathbf{f}}$$

$$\rho\Theta \frac{D\boldsymbol{\omega}}{Dt} = \boxed{\mu_t(\nabla \times \mathbf{v} - 2\boldsymbol{\omega})} + \boxed{\boldsymbol{\tau}}$$

  - Pressure
  - Transfer between linear & angular motion
  - External forces / torques

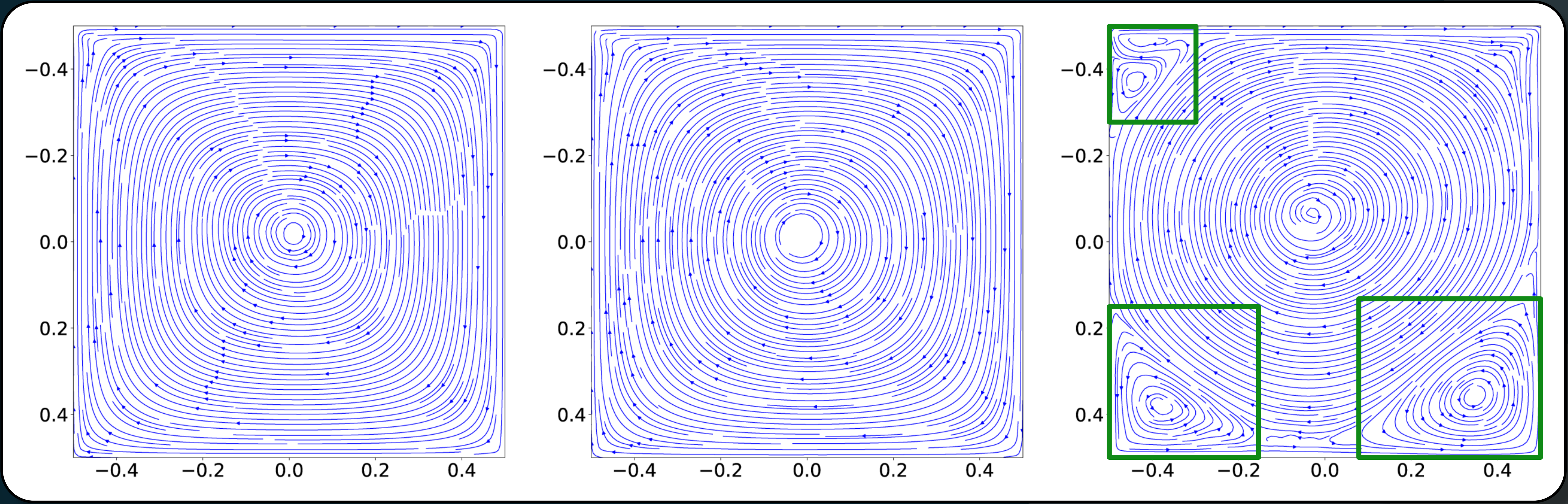
- Reduces to the Navier-Stokes equations for: $\Theta = 0,\ \mu_t = 0,\ \boldsymbol{\tau} = \mathbf{0}$
- Finally, the velocity fields should be smoothed using XSPH to ensure coherent particle motion.

Outline

- Motivation
- Vorticity Confinement
- Micropolar Model
- Results

Lid-Driven Cavity

- Set velocity to $\mathbf{v} = 1\mathrm{m/s}$ at upper boundary and no-slip at all others

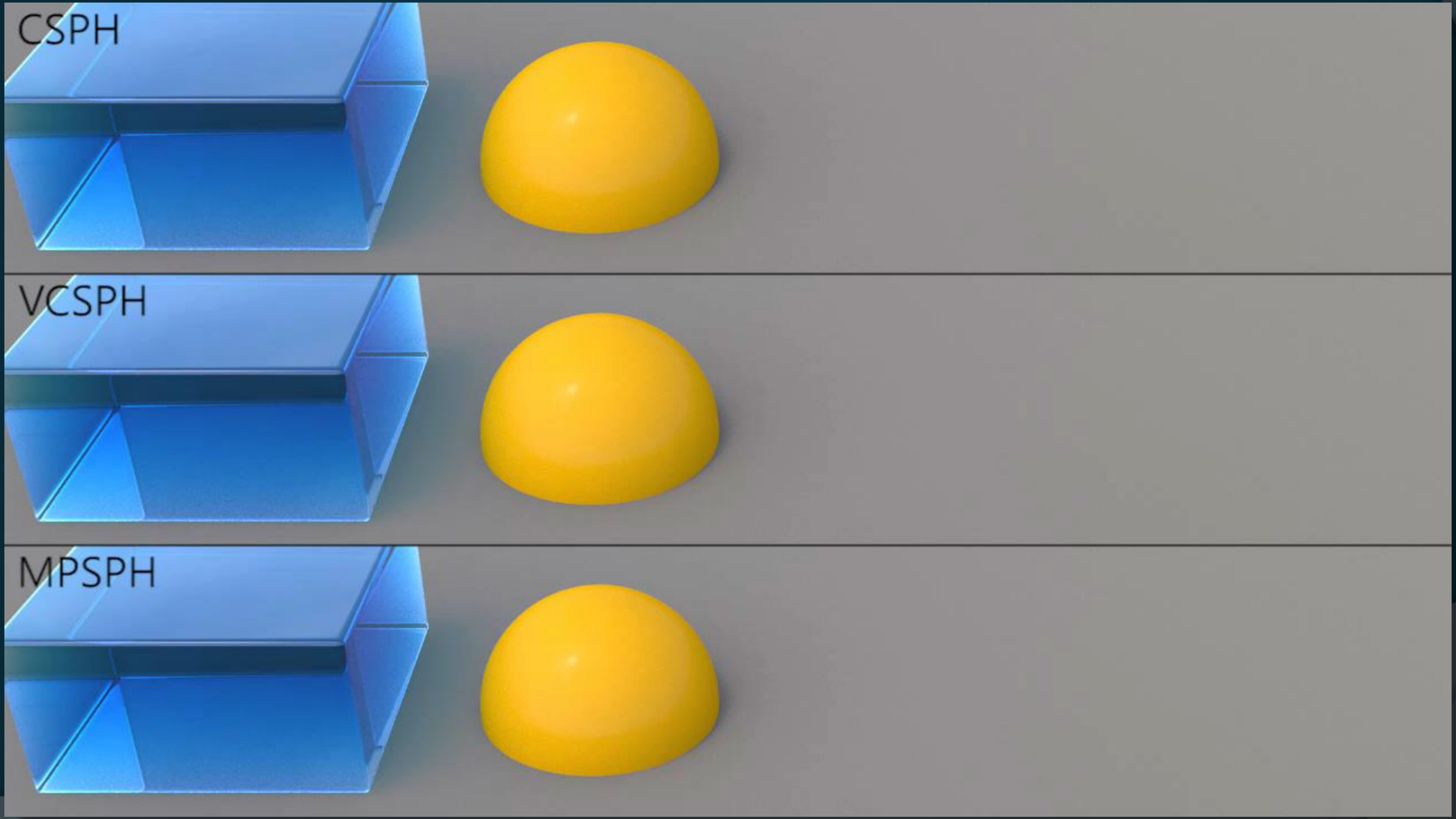


Classical SPH

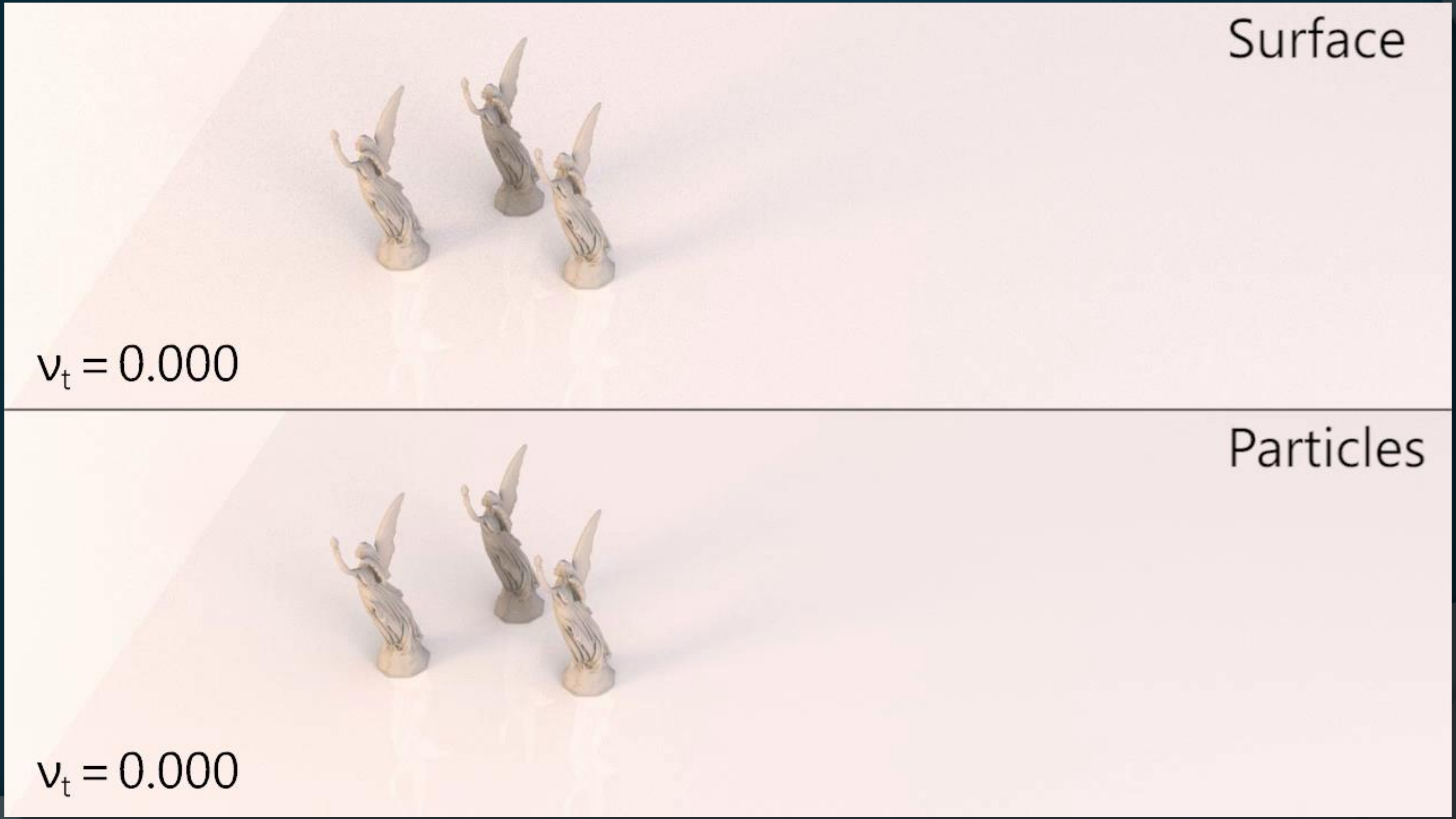
Vorticity confinement

Micropolar Model

Results



Results



Summary

- Vorticity confinement
  - Fast and easy to implement
  - Amplifies only existing vortices
  - Not momentum conserving

- Micropolar model
  - Fast and easy to implement
  - Conservation of linear/angular momentum by construction
  - Angular velocity field provides a source for new vortices